CORRECTIONS
to the paper
"COMPUTATION OF THE MITTAG-LEFFLER FUNCTION $E_{\alpha,\beta}(z)$ AND ITS DERIVATIVE"

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Professor Travis Oliphant from the Brigham Young University in Provo (Utah, USA) attracted our attention to a small misprint in the algorithm for numerical computation of the Mittag-Leffler function $E_{\alpha,\beta}(z)$(Section 5, pp. 508-509): in the 5th line from the bottom of p. 509 division by 2 was missing in the right hand side of the formula there. Furthermore Professor Oliphant proposed to move the recursion formula (2nd line from the bottom of p. 509) to the beginning of the algorithm to make it more clear and efficient. Below is the corrected version of the algorithm.

GIVEN $\alpha > 0$, $\beta \in \mathbf{R}$, $z \in \mathbf{C}$, $\rho > 0$ THEN
IF $1 < \alpha$ THEN
$\quad k_0 = \lfloor \alpha \rfloor + 1$
$\quad E_{\alpha,\beta}(z) = \frac{1}{k_0} \sum_{k=0}^{k_0-1} E_{\alpha/k_0,\beta}(z^{\frac{1}{k_0}} \exp(\frac{2\pi i k}{k_0}))$
ELSIF $z = 0$ THEN
$\quad E_{\alpha,\beta}(z) = \frac{1}{\Gamma(\beta)}$
ELSIF $|z| < 1$ THEN
$\quad k_0 = \max\{\lceil \frac{(1-\beta)}{\alpha} \rceil,\ \lceil \ln[\rho(1-|z|)]/\ln(|z|) \rceil\}$
$\quad E_{\alpha,\beta}(z) = \sum_{k=0}^{k_0} \frac{z^k}{\Gamma(\beta+\alpha k)}$

ELSIF $|z| > \lfloor 10+5\alpha \rfloor$ THEN

$k_0 = \lfloor -\ln(\rho)/\ln(|z|) \rfloor$

IF $|\arg z| < \frac{\alpha\pi}{4} + \frac{1}{2}\min\{\pi,\ \alpha\pi\}$ THEN

$E_{\alpha,\beta}(z) = \frac{1}{\alpha} z^{\frac{(1-\beta)}{\alpha}} e^{z^{1/\alpha}} - \sum_{k=1}^{k_0} \frac{z^{-k}}{\Gamma(\beta-\alpha k)}$

ELSE

$E_{\alpha,\beta}(z) = -\sum_{k=1}^{k_0} \frac{z^{-k}}{\Gamma(\beta-\alpha k)}$

ELSE

$\chi_0 = \begin{cases} \max\{1,\ 2|z|,\ (-\ln(\frac{\pi\rho}{6}))^{\alpha}\}, & \beta \geq 0 \\ \max\{(|\beta|+1)^{\alpha},\ 2|z|,\ (-2\ln(\frac{\pi\rho}{[6(|\beta|+2)(2|\beta|)^{|\beta|}]}))^{\alpha}\}, & \beta < 0 \end{cases}$

$K(\alpha,\beta,\chi,z) = \frac{1}{\alpha\pi}\chi^{\frac{(1-\beta)}{\alpha}} \exp(-\chi^{\frac{1}{\alpha}}) \frac{\chi\sin[\pi(1-\beta)] - z\sin[\pi(1-\beta+\alpha)]}{\chi^2 - 2\chi z\cos(\alpha\pi) + z^2}$

$P(\alpha,\beta,\epsilon,\phi,z) = \frac{1}{2\alpha\pi}\epsilon^{1+\frac{(1-\beta)}{\alpha}} \exp(\epsilon^{\frac{1}{\alpha}}\cos(\frac{\phi}{\alpha})) \frac{\cos(\omega)+i\sin(\omega)}{\epsilon\exp(i\phi)-z}$

$\omega = \phi(1+\frac{(1-\beta)}{\alpha}) + \epsilon^{\frac{1}{\alpha}}\sin(\frac{\phi}{\alpha})$

IF $|\arg z| > \alpha\pi$ THEN

IF $\beta \leq 1$ THEN

$E_{\alpha,\beta}(z) = \int_0^{\chi_0} K(\alpha,\beta,\chi,z)\,d\chi$

ELSE

$E_{\alpha,\beta}(z) = \int_1^{\chi_0} K(\alpha,\beta,\chi,z)\,d\chi + \int_{-\alpha\pi}^{\alpha\pi} P(\alpha,\beta,1,\phi,z)\,d\phi$

ELSIF $|\arg z| < \alpha\pi$ THEN

IF $\beta \leq 1$ THEN

$E_{\alpha,\beta}(z) = \int_0^{\chi_0} K(\alpha,\beta,\chi,z)\,d\chi + \frac{1}{\alpha} z^{\frac{(1-\beta)}{\alpha}} e^{z^{1/\alpha}}$

ELSE

$E_{\alpha,\beta}(z) = \int_{\frac{|z|}{2}}^{\chi_0} K(\alpha,\beta,\chi,z)\,d\chi + \int_{-\alpha\pi}^{\alpha\pi} P(\alpha,\beta,\frac{|z|}{2},\phi,z)\,d\phi + \frac{1}{\alpha} z^{\frac{(1-\beta)}{\alpha}} e^{z^{1/\alpha}}$

ELSE

$E_{\alpha,\beta}(z) = \int_{\frac{(|z|+1)}{2}}^{\chi_0} K(\alpha,\beta,\chi,z)\,d\chi + \int_{-\alpha\pi}^{\alpha\pi} P(\alpha,\beta,\frac{(|z|+1)}{2},\phi,z)\,d\phi$

END

The formulas for $E_{\alpha,\beta}(z)$ in this algorithm are in error at most by ρ. It is advisable to take $\rho = \varepsilon_m =$ machine precision.

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